MECH/BIOE/CEVE 454/554
COMPUTATIONAL FLUID MECHANICS

Instructor: Tayfun Tezduyar

COURSE OUTLINE:

Introductory and Quick-Use Knowledge in Computational Fluid Mechanics

Lecture Series on Computational Fluid Mechanics Chapter 1

Governing Equations and Boundary Conditions
A brief review of the governing equations of unsteady incompressible flows and the time-dependent advection-diffusion equation; introductory scale analysis; boundary conditions and computational boundary conditions

1a. Navier-Stokes Equations of Incompressible Flows and Advection-Diffusion Equation
Understanding the momentum equation and the incompressibility constraint; stress tensor; Newtonian fluids; advection-diffusion equation

1b. Scale Analysis and Boundary Conditions
Peclet and Reynolds numbers; boundary layer concept; boundary conditions; computational boundaries and boundary conditions

Lecture Series on Computational Fluid Mechanics Chapter 2

Spatial Discretization
Understanding the concept of spatial discretization for an advection-diffusion equation; how to discretize the governing equations to obtain algebraic equations or ordinary differential equations

2a. Spatial Discretization with the Finite Difference Method
One-dimensional finite differences; multi-dimensional finite differences

2b. Spatial Discretization with the Finite Element Method
Finite element shape (interpolation) functions; elements; test function; weighted residual formulation; global equation system; element-level vectors and matrices; assembly of the element-level vectors and matrices

Lecture Series on Computational Fluid Mechanics Chapter 3

Time-Integration and Related Solution Techniques
A review of time-integration in the context of finite element methods; stability and accuracy; nonlinear solution techniques; iterative solution of linear equation systems

3a. Time-Integration Techniques
Time-integration of the ordinary differential equations obtained after spatial discretization; explicit and implicit methods; predictor/multi-corrector method

3b. Nonlinear Solution Techniques and Iterative Solution of Linear Equations
Newton-Raphson method; iterative solution techniques for linear equation systems; how to compute the residual of the linear equation system; preconditioning; how to update the solution vector after each iteration

Lecture Series on Computational Fluid Mechanics Chapter 4

Spatial Discretization for Incompressible Flows
Finite element discretization of the Navier-Stokes equations of incompressible flows; weighted residual formulation; element-level vectors and matrices

4a. Finite Element Discretization of the Navier-Stokes Equations of Incompressible Flows
Weak formulation and finite element spatial discretization; obtaining the nonlinear ordinary differential equation system corresponding to the momentum equation and the algebraic constraint equation system corresponding to the incompressibility constraint

4b. Element-Level Vectors and Matrices for the Navier-Stokes Equations of Incompressible Flows
Derivation of the element-level vectors and matrices; element-level matrices for nonlinear terms; assembly of element-level vectors and matrices

Lecture Series on Computational Fluid Mechanics Chapter 5

Stabilized Formulations (for Advection-Diffusion Equation)
Understanding the stabilized formulations in the context of the time-dependent advection diffusion equation; the streamline-upwind/Petrov-Galerkin (SUPG) stabilization; extension to incompressible flows; pressure-stabilizing/Petrov-Galerkin (PSPG) formulation

5a. Stabilized Formulation for the Advection-Diffusion Equation
Element Peclet number; element Reynolds number; SUPG stabilization in 1D; stabilization parameter; SUPG stabilization in 2D and 3D

5b. Stabilized Formulation for the Navier-Stokes Equations
SUPG stabilization; PSPG stabilization; Galerkin/Least-Squares (GLS) stabilization and its relationship to SUPG and PSPG stabilizations

Lecture Series on Computational Fluid Mechanics Chapter 6

Computational Aspects of the Stabilized Formulations
The nonlinear ordinary differential equation system corresponding to the momentum equation and the algebraic constraint equation corresponding to the incompressibility constraint; derivation of the element-level vectors and matrices

6a. Semi-Discrete Forms Corresponding to the Stabilized Formulations
Semi-discrete forms; derivation of the element-level vectors; simplifications

6b. Fully Discrete Forms Corresponding to the Stabilized Formulations
Time-discretization; derivation of the element-level matrices; simplifications

Introduction to the Finite Element Method

Detailed coverage of topics in Chapters 1-3
PLUS: Element types; concepts and measures of mesh refinement
Numerical surface and volume integration

ADDITIONAL TOPICS:

Methods for flows with moving boundaries and interfaces (two-fluid or free-surface flows and fluid-structure interactions)
Comments on large-scale computing, supercomputers, parallel processing, and visualization.