

THREE-STEP EXPLICIT FINITE ELEMENT COMPUTATION OF SHALLOW WATER FLOWS ON A MASSIVELY PARALLEL COMPUTER

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SUMMARY

Massively parallel finite element strategies for large-scale computations of shallow water flows and contaminant transport are presented. The finite element discretizations, carried out on unstructured grids, are based on a three-step explicit formulation both for the shallow water equations and for the advection–diffusion equation governing the contaminant transport. Parallel implementations of these unstructured-grid-based formulations are carried out on the Army High Performance Computing Research Center Connection Machine CM-5. It is demonstrated with numerical examples that the strategies presented are applicable to large-scale computations of various shallow water flow problems.

KEY WORDS: three-step explicit scheme; parallel computing; shallow water flow; tidal flow; Tokyo Bay

INTRODUCTION

Finite element computations of shallow water flows and contaminant transport can be applied to many practical problems: design of river, coastal and offshore structures, disaster prediction and other applications related to hydrodynamic, thermal and chemical transport behaviour in oceans, lakes and rivers. In this context the finite element method is applicable to complicated water and land configurations. In practical computations of this type of problem it is essential to use methods which are as efficient and fast as the available hardware allows. Also, computations need to be carried out over long time durations to properly simulate and predict the phenomena of interest.

In recent years, massively parallel finite element computations have been successfully applied to several large-scale flow problems.^{1–3} These computations demonstrated the availability of a new level of finite element computational capability to solve practical flow problems. With the need for a high-performance computing environment to carry out simulations for practical problems in shallow water flows and contaminant transport, in this paper we present and employ a parallel explicit finite element method for computations based on unstructured grids. The finite element discretizations are based on a three-step explicit formulation both for the shallow water equations and for the advection–diffusion equation governing the contaminant transport. In these discretizations, for numerical stabilization, we use selective lumping^{4,5} for the shallow water equations and the streamline upwind/Petrov–Galerkin (SUPG) technique^{6,7} for the advection–diffusion equation. The three-node linear triangular element is

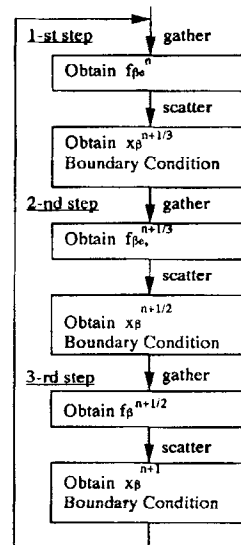


Figure 2. Structure of time step loop

NUMERICAL EXAMPLES

In order to show the efficiency of the three-step scheme, the finite element analysis for the one-dimensional wave propagation problem is considered and the computed results are compared with the results obtained by the conventional two-step scheme. Figure 3 shows the finite element discretization and boundary conditions. The numbers of elements and nodes are 500 and 378 respectively. In this computation the fluid is assumed to be a perfect fluid and the water depth is assumed to be 10 m. For the boundary conditions the incident wave amplitude and wave period are assumed to be 1.0 m and 5 s respectively. The selective lumping parameter and time increment are assumed to be 0.9 and 0.006 s respectively. Figure 4 shows the computed water elevation at $t = 30$ s. In this figure the full curve represents the computed results obtained by the three-step scheme and the broken curve represents the computed results obtained by the two-step scheme. It can be seen that the computed results are in good agreement. Figure 5 shows the computed maximum water elevation for the first 30 s. In this figure the broken line at 1 m represents the exact value. It can be seen that unstable phenomena are obtained around the inlet boundary in the case of the two-step scheme (broken curve), while the computed results obtained by the three-step scheme (full curve) are close to the exact value.

As a real-life numerical example, simulation of tidal flow and contaminant transport in Tokyo Bay is carried out. In the past, several numerical results on the tidal current in Tokyo Bay have been presented.¹⁰⁻¹² However, conventional studies have not accounted for the configuration of geometrical boundary and water depth accurately, since a coarse mesh was used in the computations. The aim of this study is to determine the details of the flow pattern of the tidal current in Tokyo Bay. Figure 6 shows the configuration of the boundary and water depth diagram of Tokyo Bay. The water depth contours are evenly spaced between 5 m and 100 m at 5 m intervals, while the maximum depth is 545 m. In this computation a fine mesh which represents the geometry accurately is employed. Figure 7 shows the finite element discretization of Tokyo Bay. The total numbers of elements and nodes are 207,797 and 107,282 respectively. This mesh is designed to keep the element Courant number constant over the entire domain.^{13,14} It can be seen that an appropriate mesh in accordance with the variation in water depth can be obtained. For the boundary condition the incident wave elevation is specified on the open boundary A-B as $\zeta = a \sin(2\pi t/T)$, where a is the incident wave amplitude, T is the incident wave

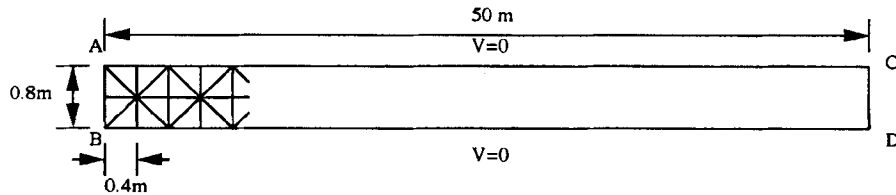


Figure 3. Finite element discretization. Boundary condition $\zeta = a \sin(2\pi t/T)$ on A-B, $U = \zeta \sqrt{g/(h + \zeta)}$ on A-B, C-D

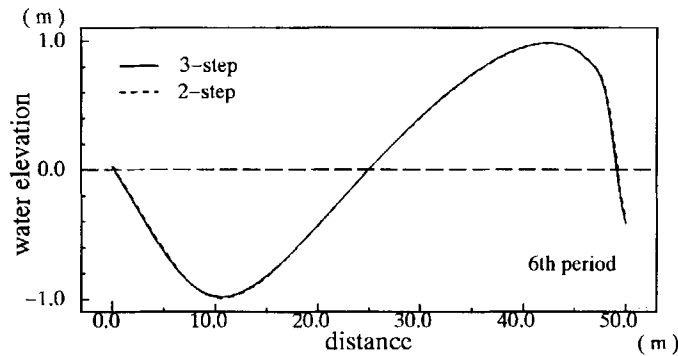


Figure 4. Computed water elevation at $t = 30$ s

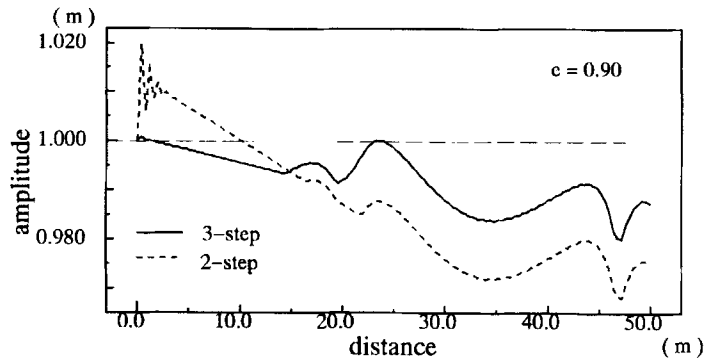


Figure 5. Computed maximum wave amplitude

period and t denotes time. The incident wave period is assumed to be 12.42 h (M_2 tide) and the incident wave amplitude is assumed to be 0.36 m.^{15,16} The non-slip boundary condition is used on boundaries. The computation is started from the state of still water. For the numerical condition the following data are used: $A_I = 5 \text{ m}^2 \text{ s}^{-1}$, $n = 0.03$ and $\kappa = 5 \text{ m}^2 \text{ s}^{-1}$. Figure 8 shows the mesh partitioning for 1024 vector units. Figures 9 and 10 show respectively the water depth diagram and finite element discretization of the Uraga-Suido channel where the configuration of water depth is highly complicated. Figures 11 and 12 show the computed current velocity distribution around Uraga-Suido at high tide ($t = 43.470$ h) and low tide ($t = 49.680$ h) respectively. Figure 13 shows the computed residual flow and Figure 14 shows the streamline of the residual flow. From the latter figure it can be seen that some small vortices exist around the Uraga-Suido channel. For the contaminant transport analysis the concentration is given at point P (see Figure 10) as an initial condition. Figure 15



Figure 6. Boundary and water depth diagram of Tokyo Bay

shows the contaminant spread at every $3 \cdot 10^5$ h interval. It can be seen that the contaminant is spread in accordance with the periodic oscillation due to the tidal current. The computational speed using 1024 vector units is 0.340 s/step and 3.16 ms/step/node.

CONCLUSIONS

A three-step explicit finite element solver has been presented and implemented on a massively parallel supercomputer. The method has been applied to several problems and the computed results are compared with the results obtained by the conventional two-step scheme. The conclusions derived are as follows.

1. The three step explicit scheme is a more stable and accurate method compared with the conventional two-step scheme. The CFL condition of the three-step explicit scheme is 1.5 times larger than that of the conventional two-step scheme.
2. The three-step explicit solver applicable to an unstructured mesh has been successfully implemented on a massively parallel supercomputer.
3. As an example a large-scale computation of the tidal current in Tokyo Bay has been carried out. The simulation also accounted for the spread of a contaminant due to tidal flows.

From the results obtained in this paper, it can be concluded that the present method can be usefully applied to large-scale computations of various shallow water flow problems.



Figure 7. Finite element discretization of Tokyo Bay

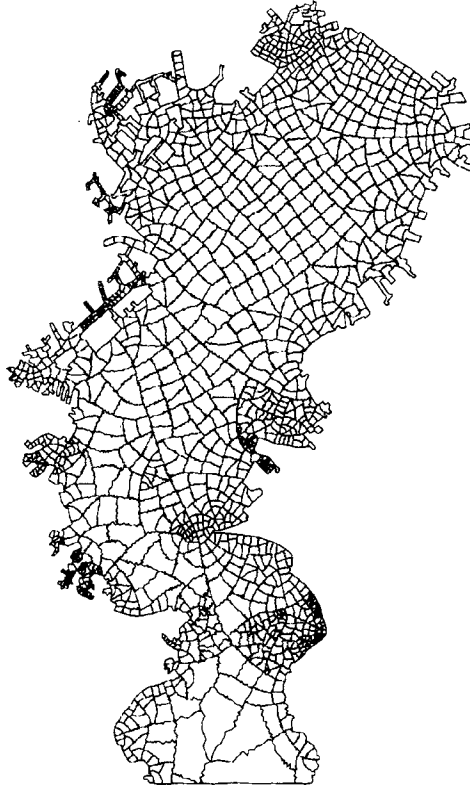


Figure 8. Mesh partitioning for 1024 vector units

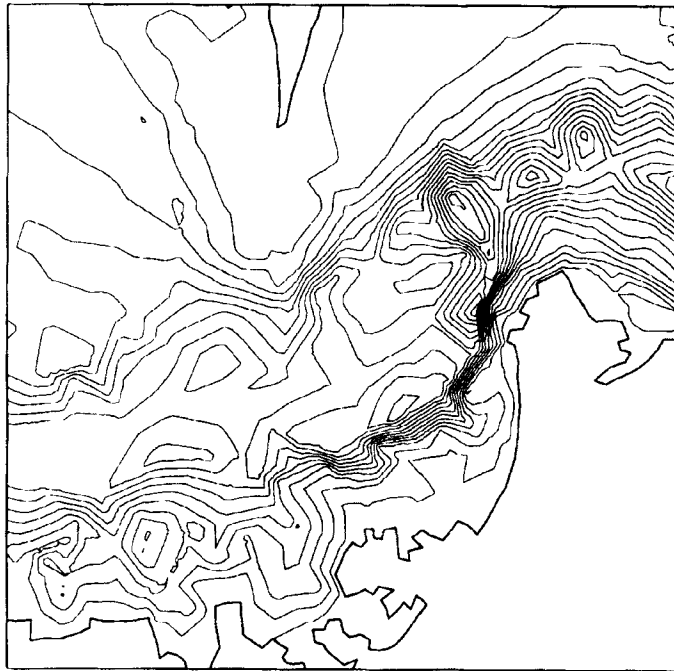


Figure 9. Boundary and water depth diagram of Uraga-Suido channel

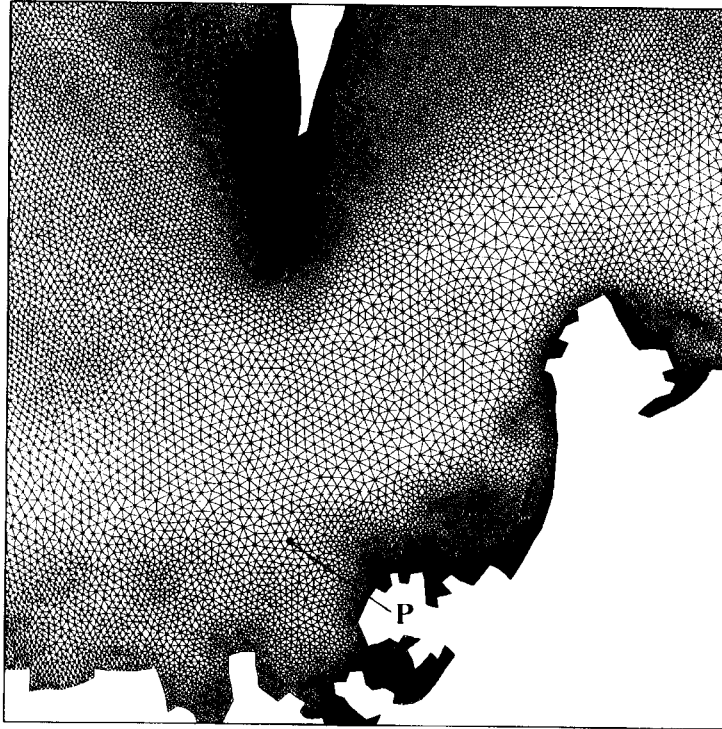


Figure 10. Finite element discretization of Uraga-Suido channel

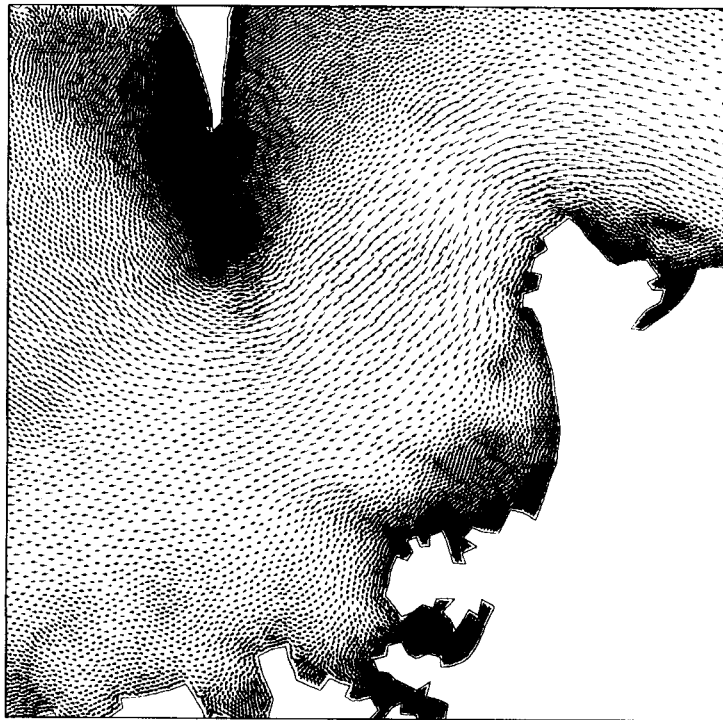


Figure 11. Computed current velocity at high tide



Figure 12. Computed current velocity at low tide

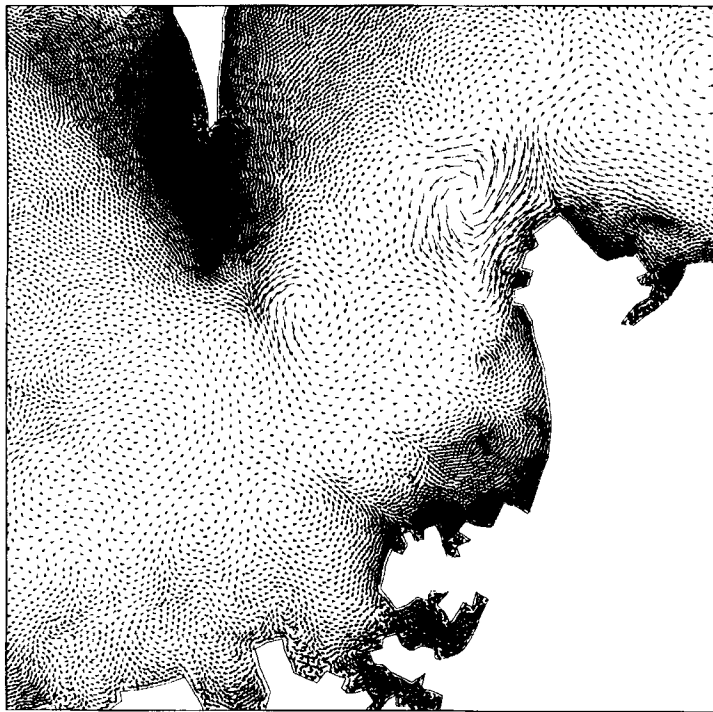


Figure 13. Computed residual current velocity



Figure 14. Streamline of residual flow

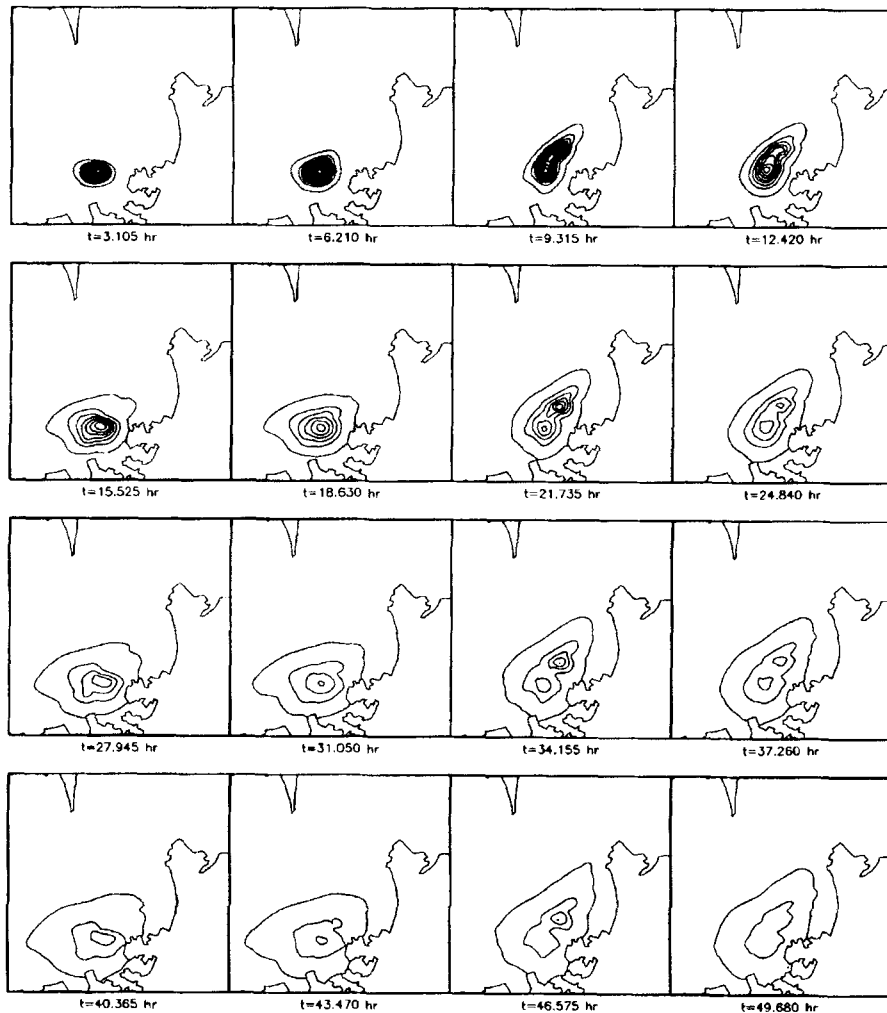


Figure 15. Computed contaminant spread at every 3.105 h

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